Antiviral 2020 Round 1 Contest

1. What is the **largest positive integer** that can be made with five 3s and **one each** of the four basic arithmetic operations (i.e. $+, -, \times, \div$)? (No brackets, exponents, concatenation, etc allowed).

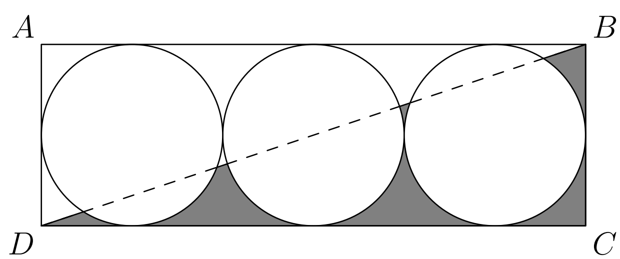
2. We normally write our numbers in **base 10**; that is, each of our 10 digits represents powers of 10, so that, for example, $157 = 7 \cdot 10^0 + 5 \cdot 10^1 + 1 \cdot 10^2$. Numbers can be written in other bases as well: computers use base 2, or binary, so each digit represents a power of 2:

$$\begin{aligned} 10011101_2 &= 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 1 \cdot 2^7 \\ &= 157 \end{aligned}$$

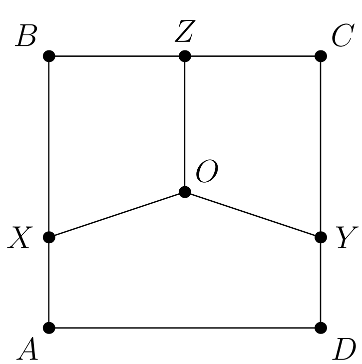
   Note that a subscript 2 is used to remind us that this number is in **base 2**.

   What is the one-hundredth smallest positive integer that can be written using no digits other than 0, 1, and 2 in base 4?

3. Three identical circles of radius 2 are externally tangent and inscribed within a rectangle of length 12 and width 4. Note that the shaded region represents the the portion of triangle BCD that does not overlap with the circles. The area of this shaded region can be represented as $m - n\pi$, where m and n are positive integers. Find $m + n$.



4. In the diagram below (diagram not to scale), $ABCD$ is a square, Z is the midpoint of BC, $OZ \perp BC$ and $OZ = \frac{1}{2}AB$. X and Y are on AB and CD, respectively, such that $AX = DY$. It is known that the three enclosed regions $ADYOX, BXOZ, CZOY$ have the same area. The value of $\frac{AX}{XB}$ can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



5. How many tetrahedra with positive volume can be made using 4 distinct vertices of a cube?

6. Let a and b be positive integers, such that $\log_{\sqrt{2}} a$ is rational. If

$$\log_{\sqrt{2}} a + \log_a b = \frac{\log_{\sqrt{2}} b}{2},$$

   then compute the sum of all possible values of $a + b$.