Antiviral 2020 Round 2 Contest

1. A **chameleon** is a 4-digit positive integer with distinct digits (and no leading zeroes). What is the smallest chameleon that is divisible by 9?

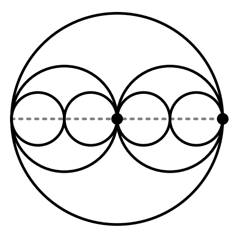
2. Trish has four types of socks in her drawer, in the following quantities: 3 blue polka-dot, 5 red polka-dot, 4 blue striped, and 3 red striped. She's not too fussy about matching: she'll wear any two socks as long as they're both blue **or** both polka-dot. If Trish has to pick her socks in the dark (and thus never knows what socks she has picked out), how many will she have to take out before she's sure she has a pair that she can wear?

3. a, b, and c are three positive integers such that:

   - $a, b, c \geq 15$;
   - a and b have no common factors (other than 1);
   - $a^2 + b^2 = c^2$ (i.e. a, b and c are a **Pythagorean triple**);
   - $c = b + 2$.

   What is the **least** possible value of c?

4. Starting with a circle of radius 4, we inscribe a two circles of radius 2 such that the diameters of the two small circles lie on the same line. We then inscribe two circles in each of the 2 circles in a similar manner. We continue this process indefinitely, and find that the sum of the areas of the circles we draw is $k\pi$ for some integer k. What is the value of k?



5. You are given a 5 by 5 grid. You place the numbers 1 to 25 in each cell such that for every cell, if there exists a cell below it, then the number in that cell is greater than that of the original cell. Likewise, if there exists a cell to the right of it, then that cell has a number greater than that of the original cell. Out of all the possible placements of the 25 numbers, how many distinct cells can the number 18 be placed in?
   For example, 18 can be placed in the cell at the intersection of the 4th row and 4th column as follows:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	19
15	16	17	18	20
21	22	23	24	25

   In contrast, the following arrangement does not work:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
18	19	17	16	20
21	22	23	24	25

6. We consider a modified *Game of Life*. We begin with a certain amount of live cells on an infinite square grid, and every turn the following modifications are implemented:

   (a) If a cell is **orthogonally** adjacent (a.k.a. shares a side) to **at least two** live cells, then it becomes live in the next turn.

   (b) All other cells become dead in the next turn.

   For how many positive integers n at most 1970, does there exist an initial configuration of n live cells such that this configuration stays constant after each turn (no cells are ever changed from live to dead or vice versa)?