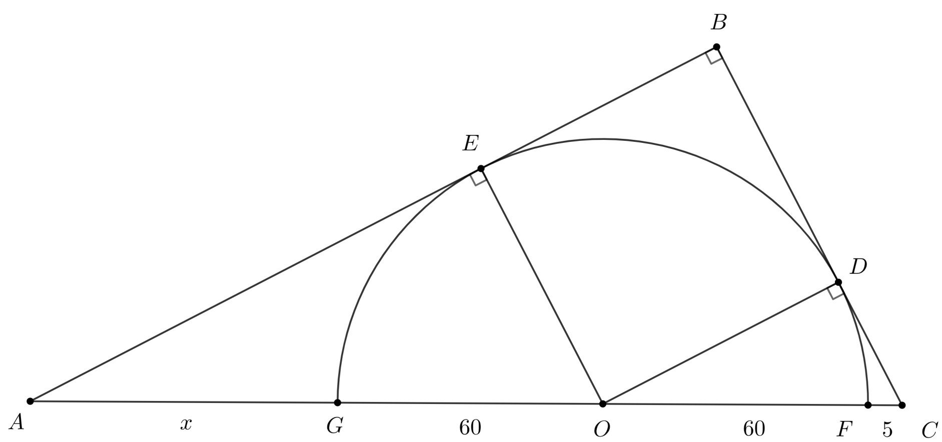
Antiviral 2020 Round 3 Contest

1. Glenn bought a collection of Superman figurines. Each individual figurine costs the same amount of money, and Glenn paid \$224 in total. Unfortunately, one of the figurines has been chewed up by his dog, so he sold all of the remaining figurines for \$4 more each than he originally paid for them. Glenn managed to break even — he got back \$224. How many figurines did he originally buy?

2. If $x^3 + 9x^2 + 3x + 3 = 1933$, what is the value of $5x^4 + 3x^2 + 9x + 8$?

3. The value of
$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{50^2}\right)$$
can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

4. In the diagram below (which is not to scale), GF is the diameter of a semicircle centered at O. The semicircle is tangent to segment AB at E and BC at D. Triangle ABC is right-angled at B and A, G, O, F and C all lie on the same line. We have $GO = OF = 60$ and $FC = 5$. Find the length of AG, shown as x in the diagram.



5. 13 wizards stand in a circle. 5 of these wizards are evil wizards, while the other 8 are non-evil. The evil wizards all simultaneously fire laser beams at some other randomly-chosen wizard. Each of the evil wizards may choose any other wizard, evil or non-evil, and two or more evil wizards may fire their lasers at the same other wizard. Two wizards may shoot each other, but wizards do not shoot themselves.

   The probability that 3 of the wizards form the vertices of a laser-beam triangle can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

6. Real numbers a and b are chosen uniformly at random from the interval $[0, 1]$.

   The probability that
$$\left\lfloor \log_2\left(\frac{1}{a+b}\right)\right\rfloor$$
is odd can be written as $\frac{m}{n}$ where m and n are relatively prime integers. Find $m + n$.