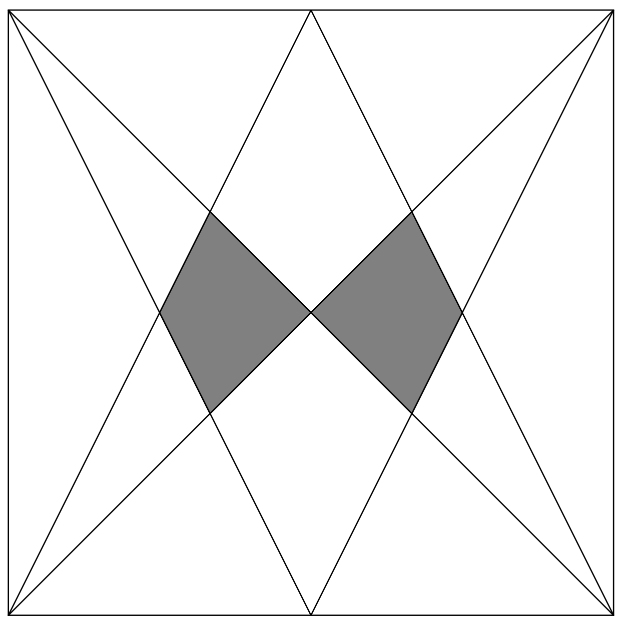
Antiviral 2020 Round 4 Contest

1. Let a and b be positive integers that have a product of 2019. Compute:

$$(-1)^a \times (-1)^b$$

2. A positive integer is *prime-o* if it has an odd, prime number of factors. How many positive integers less than 100 are *prime-o*?

3. In the following square, the shaded region is formed by the intersection of lines joining vertices and midpoints. The shaded region consists of $S\%$ of the entire square. What is the value of $\lfloor S \rfloor$, the greatest integer less than or equal to S?



4. The variables x and y are chosen uniformly at random from the interval $[0, 1]$. The probability that both $x^2 + y^2 < 1$ and $(1 - x)^2 + (1 - y)^2 < 1$ are satisfied is p. Find $\lfloor 1000p \rfloor$, which is the greatest integer less than or equal to $1000p$.

5. Spirited Sam wrote down two arithmetic sequences on the board. Pensive Pam computed the product of corresponding terms of the arithmetic sequences. Careless Cam erased everything except for the first three terms that Pam wrote, which were 310, 455, 624. What was the sixth term in Pam's sequence?

6. The sequence

$$\sum_{i=1}^{2019} \tan(i) \times \tan(i + 1) = \tan(1) \times \tan(2) + \tan(2) \times \tan(3) + \ldots + \tan(2019) \times \tan(2020)$$

can be expressed in the form $\frac{\tan(x)}{\tan(y)} - z$, where x, y, and z are positive integers. If all given angles are in radians, determine $x + y + z$.